Re-Exam

Spring 2021

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!
60 points

1. Short questions (10 points total)

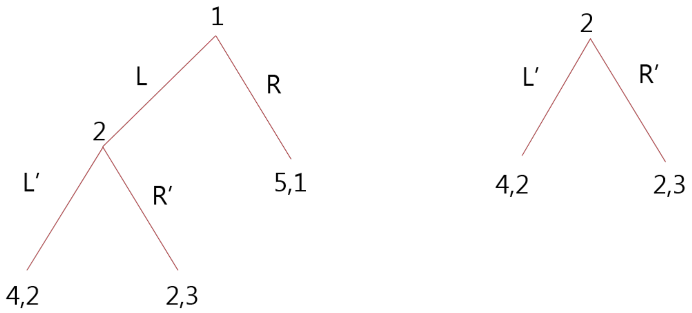
   (a) What are the four criteria for something to be a costly signal (as for example, white shoes signaling wealth). (4 points)

   Solution: 1. The signal needs to be costly for both types. 2. The signal is more affordable for those with a desirable, at least partly hidden type 3. Those with the desirable trait are more likely to send the signal 4. Those who exhibit the signal are more desirable (e.g. more often chosen as friends or mates) to those who care about the desirable trait.

   (b) Elisa and Sofie play a game called "Race to 100". Elisa goes first and then the two players take turn choosing numbers between 1 and 9. On each turn they add the new number to a running total. The player who brings the number to exactely 100 wins the game. If both players play optimally, who will win the game? Does this game have a first mover advantage? Explain your reasoning. (3 points)

   Solution: Elisa will win if the running total is at least 91 on her turn, because she can bring the total to exactly 100 from any number between 91 and 99. However, if the running total is 90, then Elisa cannot win because she cannot get the total to 100 and after her turn the total will be at least 91. So, any player faced with a total of 90 on her turn will lose the game. This also means that Elisa will win if the running total is at least 81 (and no higher than 89) on her turn, because she can bring the total to 90 and force Sofie to put the total at 91 or higher. Continue this process all the way back to the start of the game. Elisa will win if the running total is at least 1 (and no higher than 9) on her turn. This is guaranteed to happen if she goes second so the game has a second-mover advantage. The second player can take the running total to a multiple of 10 on each turn and is guaranteed to be able to win.

   (c) In Lecture 12, we talked about that people don't always act in a payoff maximizing way. They might be maximizing utility, with part of their utility comining from their beliefs and the beliefs of others. Have a look at the following two games. a) What is the SPNE of each game if players only maximize their montary payoff? b) Now assume that each game is played by 100 people. Assume that the players have reciprocal preferences. They do not only care about their monetary payoff. They also care about reciprocity. In which game (left or right) should we empirically observe the higher share of player 2's choosing L', if they get to play? Explain in 2-3 sentences. (3 points)



Solution: The SPNE for the left game is R,R'. For the right game it is R'. If they have reciprocal preferences we should observe more player 2's chose L' in the first game (left) than the second game. If players have reciprocal preferences this means that they want to reward player 1 being nice to them (giving up payoff to increase the payoff of player 2). Thus, they will reward player 1 by choosing the outcome that is better for player 1.

2. Three buyers compete in a sealed-bid auction. Their valuations of the object are known to be distributed independently and uniformly on [0, 1]. It so happened that their actual values are v1 = .7; v2 = .2; v3 = .9, but everybody only knows her own value. Assume that bidders are risk-neutral. (9 points total)

   (a) Using the appropriate formulas determine what bids will be submitted, who will win the auction at which price and what will be the seller's revenue if the auction mechanism is:

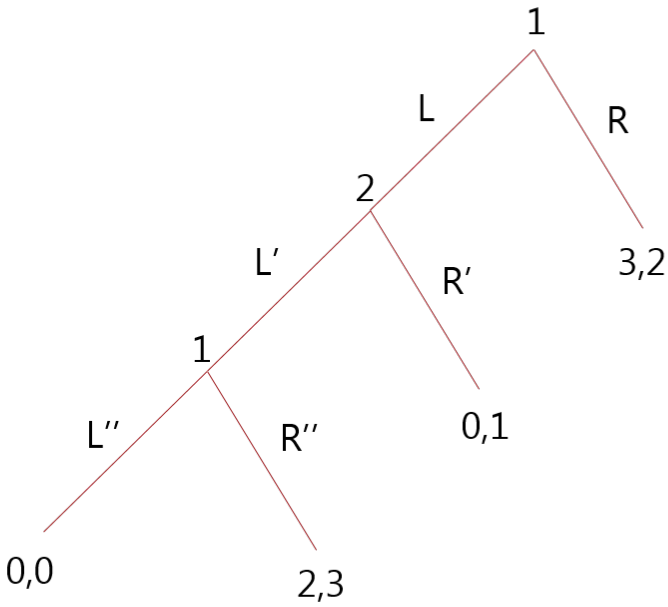
   - first-price
   - second-price
   - second-price with reserve price .8 (reserve price=minimum price that the seller requires to give up the object)

   Solution: Player 3 will win in every case. Under first-price, given risk neutrality and independent values of n players, uniformly distributed on [0, 1] the formula is $b_i = \frac{n-1}{n} v_i$ so the winning bid b3 and hence seller's revenue will be .6.
   Under second-price, truth-telling ($b_i = v_i$) is weakly dominant so the revenue will be .7 with no reserve price and .8 when reserve price is there. (7 points)

   (b) What problem can occur in real life auctions when the auction is a sealed-bid first price auction with common values? Why does this happen? Why would this not happen if the players played the BNE? 2-3 sentences.

   Solution: In a common value auction there is a risk of the winner's curse. The winner's curse arises when a bidder fails to take account of the fact that, when he wins, he is likely to have made an overly optimistic estimate of the object's value. If bidders receive signals about the value of a good, there is a chance that some will receive very high signals. If they act on them, they will have paid too much. In the BNE they bid more aggressively on their own signal. (2 points)

3. Consider the following game written in extensive form (11 points total):



(a) Is this a game of complete or incomplete information? (1 point)

> **Solution:** This is a game of complete information

(b) Write down the strategy sets for Player 1 and Player 2, and find all pure strategy Subgame Perfect Nash Equilibria. (6 points)

> **Solution:** $S_1 = LL'', LR'', RL'', RR'', S_2 = L', R'$.
> The unique Subgame Perfect Nash Equilibrium is RR", L'. which can be found by backwards induction. Player 1 will choose R" if he reaches his second node since $2 > 0$. This means Player 2 will choose L' if she reaches her decision node, since 3>1. This means Player 1 will choose R at his initial node since $3 > 2$.

(c) Find all pure strategy Nash equilibria of this game. (4 points)

> **Solution:** Write the normal form of this game:

		Player 2	
		S	W
Player 1	S	$0, 0$	$0, 1$
	W	$2, 3$	$0, 1$
	S	$3, 2$	$3, 2$
	W	$3, 2$	$3, 2$

> The pure strategy Nash Equilibria are (RL",L'), (RR",L'), (RL",R'), (RR",R').

4. Consider a two player game between FastBike and EasyBike. Each of them produces and sells electric bicycles. Each firm can set either a high or a low price for their standard model bicycle. If they both set a high price, each receives profits of $64,000 per year. If one sets a low price and the other sets a high price, the low price firm earns profits of $72,000 per year while the high-price firm earns $20,000. If they both set a low price each firm receives profits of $57,000. (22 total)

   (a) Verify that this game has a prisoner's dilemma structure by looking at the ranking of the payoffs associated with the different strategy combinations. What are the equilibrium strategies and payoffs in the simultaneous-move game if the players make the price decisions only once? (6 points)

Solution: The payoffs are ranked as follows: high payoff from cheating (72) > cooperative payoff (64) > defect payoff (57) > low payoff from cooperating (20). This conforms to the pattern in the text so the game is a prisoners' dilemma. If the game is played once, the Nash equilibrium strategies are (Low, Low) and payoffs are (57, 57).

(b) If the firms decide to play this game for a fixed number of one-year periods, say for 4 years, what will each firm's total profit be at the end of the game (don't discount). Explain how you arrived at your answers. (4 points)

Solution: Total profits at the end of four years = 4 x 57 = 228. Firms know that the game ends in four years so they can look forward to the end of the game and use rollback to find that it's best to cheat in year 4. Similarly, it is best to cheat in each preceding year as well. It follows that it is not possible to sustain cooperation in the finite game.

(c) Suppose that the firms play the game repeatedly forever. Let each of them use a grim strategy in which they both price high unless one of them defects, in which they price low for the rest of the game. What is the one time gain from defecting against an opponent playing such a strategy? How much does each firm lose, in each future period, after it defects once? If $\delta = 0.8$, will it be worthwhile for them to cooperate? Find the range of values for δ for which this strategy is able to sustain cooperation between the two firms. (5 points)

Solution: The one-time gain from defecting = 72 - 64 = 8. Loss in every future period = 64 - 57 = 7. Cheating is beneficial here if the gain exceeds the present discounted value of future losses or if 8 > 7/r. Thus, r > 7/8 (or $\delta < 8/15$) makes cheating worthwhile, and r < 7/8 lets the grim strategy sustain cooperation between the firms in the infinite version of the game. If $\delta = 0.8$, cooperation can be sustained.

(d) Suppose the firms play this game repeatedly year after year, with neither expecting any change in their interaction. Surprisingly, the world ends after four years with neither firm anticipating this. How is this situation different and how is it similar to c)? (3 points)

Solution: With no known end of the world, the firms can sustain cooperation if r < 7/8 ($\delta > 8/15$) as in part (c). This is different from part (b) because the firms see no fixed end point of the game and can't use backward induction. Instead, they assume the game is infinite and use the grim strategy to sustain cooperative outcome.

(e) Suppose now that there is a 10% probability that one of them will go bankrupt in any given year. If bankruptcy occurs, the repeated game between the firms end. Will this knowledge change the firms' actions when $\delta = 0.8$? What if the probability of a bankruptcy increases to 35% in any given year? (4 points)

Solution: Now we have to calculate the chance that the game continues times the discount factor $p * \delta$. 10% probability of bankruptcy translates into a 90% probability that the game continues, so p = 0.9. Then, for r = 0.25 (d = 0.8), the discount factor is 0.72. This is still larger than 8/15, so the firms will still cooperate in this case. For a 35% probability of bankruptcy, p = 0.65, so if bankruptcy becomes more certain, cheating becomes worthwhile $(0,52 < 8/15)$.

5. Suppose electricians come in two types, competent and incompetent. Both types of electricians can get certified, but for the incompetent types, certification takes extra time and effort. Competent ones have to spend C months preparing for the certification exam. Incompetent ones take twice as long. Certified electricians can earn 100 (thousand DKK) each year working for professional contractors, uncertified ones can only earn 25 (thousand DKK) working freelance. Uncertified ones won't get hired by professional contractors. Each type of electrician gets a payoff of $\sqrt{S} - M$ (Typo in original exam), where S is the salary measured in thousands of kroners and M is the number of months getting certified. What is the range of values of C for which a competent electrician will choose to get certification to show his type, but an incompetent one will not? (Tip: No need to write a game tree to answer this) (8 points)

Solution: A competent electrician's payoff after obtaining the certification is $\sqrt{100} - C$ and in the absence of the certification, this electrician's payoff is $\sqrt{25}$. The competent electrician will invest in the certificate as long as $\sqrt{100} - C \leq \sqrt{25}$, or as long as $10 - C \leq 5$, or as long as $5 \leq C$. The incompetent electrician earns only $\sqrt{100} - 2C$ after certification versus $\sqrt{25}$ without certification. He invests in the certificate only if $\sqrt{100} - 2C \leq \sqrt{25}$, or as long as $10 - 2C \leq 5$, or as long as $5/2 \leq C$. The range of values of C for which the competent electrician gets the certificate but the incompetent electrician does not is $5 \leq C \leq 2.5$.